

The Final Values of the Coefficients in the New Lunar Theory.

By Ernest W. Brown, Sc.D., F.R.S.

1. As has been stated on previous occasions the problem under consideration, and now completed, is that of Delaunay's theory, with the additional terms introduced by replacing a/a' by $a(E-M)/a'(E+M)$. In earlier papers* I have given a general account of the methods used and of the means taken to secure accuracy, with some indication of the extent to which the results conform with those deduced from observation. The main object of the present communication is to give the complete numerical values of the coefficients of all periodic terms in longitude and latitude which are as great as $0''.01$, and in parallax those which are as great as $0''.001$. Every coefficient has been taken to at least one more place in the computations.

A secondary object is to compare the results with those of Hansen, so as to show explicitly the extent of the agreement between the two theories. The results of Delaunay may be used as a check where differences from those of Hansen occur; but slow convergence makes so many of Delaunay's coefficients doubtful that it does not seem useful to give them here. They can, if necessary, be directly obtained from Newcomb's "Transformation of Hansen's Theory,"† the values which he there finds for the latter being inserted in the tables below. The general result of this comparison, stated briefly, is that where sensible differences occur between my results and those of Hansen my values are confirmed by Delaunay when allowance is made for the slow convergence of his series.

The mean motions of the perigee and node are not given here. Several comparisons with the results of Delaunay have been made on previous occasions, and the final numbers are fully set out and discussed in a paper "On the Degree of Accuracy of the New Lunar Theory and on the Final Values of the Mean Motions of the Perigee and Node."‡

* In particular those in the *Monthly Notices* for April 1904 and December 1904.

† *Washington Astr. Papers*, vol. i. pt. (ii.)

‡ *Monthly Notices*, 1904 April, pp. 524–534. I am much indebted to Professor H. L. Rice of the Naval Observatory, Washington, for pointing out an error in the paragraph numbered 5 in this paper. The expressions

$$-\frac{3\delta a'}{a'} \cdot \frac{0''.70}{328243}, + \frac{3\delta a'}{a'} \cdot \frac{0''.20}{328243}$$

should be replaced by

$$-328243 \frac{3\delta a'}{a'} 0''.70, + 328243 \frac{3\delta a'}{a'} 0''.20,$$

and all the numbers in the second and third columns of the table headed "Indirect Planetary Action" (p. 529) should be multiplied by 1.077. Fortunately the changes in the final results are almost insensible: $-0''.01$ is to be added to the calculated results for the perigee given on pp. 525, 532, and $+0''.01$ to the calculated results for the node on the same pages.

2. *The Constants of the Theory.*—The numerical values of the constants used in reducing the theory to numbers are as follows :

MOON (1850.0).	SUN (1850.0).
$n = 17\ 325\ 594''\cdot 06$	$n' = 1\ 295\ 977''\cdot 415$
$e = \cdot 054\ 900\ 56$	$e' = \cdot 016\ 771\ 91$
$\gamma = \cdot 044\ 887\ 16$	$\gamma' = 0$
$\frac{1}{a} = 3412''\cdot 596$	$\frac{1}{a'} = 8''\cdot 7800$
$\frac{E}{M} = 81\cdot 500.$	

The definitions of these constants are those adopted by Delaunay.

The object here being the comparison of the two sets of *theoretical* coefficients and not the comparison of either with the observed values, Hansen's coefficients are given directly from Newcomb's results, referred to above, and the changes which they require when Hansen's constants are altered to the values just given are shown separately. The comparison of the values for e, γ, a is seen directly from the coefficients of $\sin l$ and $\sin F$ and $\cos o$ in the longitude, latitude, and parallax respectively. Hansen's $e' = \cdot 016\ 792\ 28$, and his $1/a' = 8''\cdot 848$. His values for n, n' are the same as mine within the limits of accuracy of the comparison.*

3. *Explanation of the Tables Below.*—The *first* column gives the principal characteristic (C) of each set of terms. In the *second, third, fourth, and fifth* columns are the multiples of l, l', F, D (Delaunay's notation), which enter into the arguments ; the characteristic and multiples of l, l', F being the same for each set of terms (that is, for those terms whose arguments differ only by multiples of $2D$), they are only set down for the first term of each set. The *sixth* column (headed B) contains my final values. The *seventh* column (headed H) contains Hansen's theoretical values, with his constants. The *eighth* column gives the reduction (R) necessary to reduce Hansen's results to my set of constants. The *last* column (B—H—R) shows the real differences between the results of the two theories. The coefficients to which letters are attached are discussed in the following section.

4. *Observations on the Results.*—As has been stated the calculations were constructed so as to include all coefficients in longitude and parallax as great as $0''\cdot 01$, and to neglect all characteristics which did not have at least one coefficient as large as this amount. But since the calculations in each characteristic included all coefficients of $0''\cdot 0005$ and over, there are com-

* A typographical error occurs on p. 526, where Newcomb's coefficient for the principal elliptic inequality is set down as $22659''\cdot 58$; it should, of course, be $22639''\cdot 58$.

paratively few lying between $0''.005$ and $0''.01$ (which are of course entered as equal to $0''.01$) present in Hansen's theory and not in mine. In longitude there are but two, and for one of these, characteristic e^6 and argument $6l$, the elliptic value suffices; the other ($0''.01$) has the characteristic $e^3e'\gamma^2$, and Hansen and Delaunay agree on its value. In latitude there are two, with characteristic $\gamma e^4e'$, to which the previous remark also applies. For the parallax similar remarks may be made with the degree of accuracy $0''.001$ at the start; there are three such coefficients, characteristic e^4e' , each having a coefficient, according to Hansen, of $0''.001$, and they are probably less than $0''.001$ and greater than $0''.0005$. These are, of course, quite unimportant from a practical point of view, and they really only need consideration when the number of them is comparatively large. It is, therefore, of some interest to know the sum of the absolute values of the differences B—H—R in each coordinate to obtain an idea of the maximum differences which tables constructed on the two theories would show. Adding the numbers in the columns B—H—R, without regard to sign, we obtain

In longitude	3'61
In latitude	1'90
In parallax	'242

The great majority of these differences are in the coefficients of short-period terms. Also, there are few which exceed $0''.02$ in longitude and latitude and $0''.002$ in parallax.

Certain coefficients require special mention. In that of the *variation*, marked (a) below, there is a difference of $0''.16$. Newcomb's comparison shows a practical agreement between Hansen and Delaunay. In an earlier paper* I stated that Delaunay's value for the part of this coefficient with characteristic e^2 was in error by about $0''.10$, and the correction has been confirmed by M. Andoyer.† This accounts for all but $0''.06$ of the difference between my result and that of Delaunay, and an estimate for neglected terms in his series must be uncertain by at least this amount.

In the *parallactic inequality*, marked (b), there is a difference of $-0''.32$. I have given‡ a comparison between my results and those of Delaunay for the various parts of this coefficient. This comparison showed that $-0''.21$ must be added to the latter to account for neglected terms; an estimate for such terms

* *Monthly Notices*, vol. lvii. (1897), p. 336.

† *Comptes Rendus*, vol. cxxx. (1900), p. 1532.

‡ *Monthly Notices*, April 1904, p. 534. My calculated results only included the terms of characteristics of orders 1, 3. Those whose characteristics were of order 5 were estimated from Delaunay's series to amount to $+0''.02 \pm 0''.02$. Complete calculations for the latter have since shown that these terms give $+0''.024$, so that the results of that paper are correct to the degree of accuracy there adopted.

confirms my value within the limits of possible error of the estimate.

In the coefficients, marked (c), the differences are $0''.23$ in a coefficient $1''.09$, and $0''.21$ in a coefficient $1''.30$. Newcomb estimates $0''.87$ and $1''.39$ as Delaunay's complete values for the respective coefficients, but any estimates must be uncertain by at least the differences between the two sets of results. The periods are long and the coefficients are difficult to determine by any method in which approximation along powers of m is used. Even with the method of this theory the loss of accuracy owing to small divisors is so great that these two coefficients to a certain extent determine the number of places of decimals to be adopted at the outset of the whole work.*

It is not evident why the differences marked (f), which depend mainly on γ , practically disappear if we adopt $18461''.5$ instead of $18463''.3$ for the principal term in latitude of Hansen's theory, other differences being not materially affected. It would almost appear as though a value near the former was really the constant of Hansen's theory, especially as these coefficients are easy to determine accurately.

True Longitude—Mean Longitude. Coefficients of Sines.

O.	<i>l.</i>	<i>l'.</i>	F.	D.	B.	H.	R.	B-(H+R).
I	0	0	0	6	+ 0''13	+ '13
				4	+ 13'90	+ 13'90
				2	+ 2369'90	+ 2369'75	- '01	+ '16 (a)
e	1	0	0	6	+ '02	+ '02
				4	+ 1'98	+ 1'98
				2	+ 191'95	+ 191'95
				0	+ 22639'58	+ 22640'15	- '57	...
				-2	- 4586'44	- 4586'56	+ '11	+ '01
				-4	- 38'43	- 38'43
				-6	- '39	- '40	...	+ '01
e'	0	1	0	4	- '29	- '29
				2	- 24'45	- 24'45	+ '03	- '03
				0	- 668'94	- 669'85	+ '81	+ '10
				-2	- 165'35	- 165'22	+ '20	- '03
				-4	- 1'88	- 1'89	...	+ '01
				-6	- '02	- '02
a	0	0	0	5	'00	+ '01	...	- '01
				3	+ '40	+ '41	...	- '01
				1	- 124'79	- 125'43	+ '96	- '32 (b)

* See a paper by the writer, "On the Small Divisors in the Lunar Theory," *Trans. Amer. Math. Soc.* vol. iii. (1902), pp. 159-185. The transformations given in sect. iii. of the paper were unfortunately only worked out after most of the calculations had been completed, so that they have not been used for computation.

	True Longitude—Mean Longitude.					Coefficients of Sines.				
C.	L.	V.	F.	D.	B.	H.	R.	B—(H+R).		
e^2	2	0	0	4	+	"21	+	'22	...	—'01
				2	+	14'39	+	14'38	...	+ '01
				0	+	769'02	+	769'06	—'04	...
				—2	—	211'66	—	211'71	+ '01	+ '04
				—4	—	30'77	—	30'78	...	+ '01
				—6	—	'57	—	'57
				—8	—	'01	—'01
ee'	1	1	0	4	—	'05	—	'05
				2	—	2'93	—	2'93
				0	—	109'80	—	109'92	+ '14	—'02
				—2	—	206'22	—	206'46	+ '26	—'02
				—4	—	4'40	—	4'41	+ '01	...
				—6	—	'07	—	'07
	1—1	0	6	+	'01	+ '01
				4	+	'28	+	'29	...	—'01
				2	+	14'60	+	14'60	—'02	+ '02
				0	+	147'88	+	148'02	—'18	+ '04
				—2	+	28'51	+	28'56	—'03	—'02
				—4	+	'64	+	'64
				—6	+	'01	+	'01
e'^2	0	2	0	2	—	'19	—	'19
				0	—	7'51	—	7'51	+ '02	—'02
				—2	—	8'12	—	8'13	+ '02	—'01
				—4	—	'15	—	'15
γ^2	0	0	2	4	—	'09	—	'09
				2	—	5'74	—	5'74
				0	—	411'61	—	411'60	+ '08	—'09(<i>f</i>)
				—2	—	55'17	—	55'25	+ '01	+ '07
				—4	+	'03	+ '03
ea	1	0	0	1	—	8'44	—	8'48	+ '07	—'03
				—1	+	18'55	+	18'70	—'14	—'01
				—3	+	3'21	+	3'23	—'02	...
				—5	+	'01	+	'01
$e'\alpha$	0	1	0	3	+	'15	+	'15
				1	+	17'99	+	18'09	—'16	+ '06
				—1	+	'56	+	'60	...	—'04
				—3	—	'07	—	'06	...	—'01

True Longitude—Mean Longitude.					Coefficients of Sines.					
O.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).		
e3	3	0	0	4	+	"02	+	'02
				2	+	1'06	+	1'06
				0	+	36'12	+	36'13	...	—'01
				—2	—	13'19	—	13'19
				—4	—	1'19	—	1'18	...	—'01
				—6	—	'29	—	'29
				—8	—	'01	—'01
e2e'	2	1	0	4	—	'01	—'01
				2	—	'29	—	'29
				0	—	7'66	—	7'67	+ '01	...
				—2	—	8'64	—	8'66	+ '01	+ '01
				—4	—	2'74	—	2'75	...	+ '01
				—6	—	'09	—	'09
				—8	—	'01	—'01
2-1	0	4	+	'03	+	'03
				2	+	1'18	+	1'18
				0	+	9'72	+	9'72	—'01	+ '01
				—2	—	2'50	—	2'52	...	+ '02
				—4	+	'36	+	'36
				—6	+	'01	+	'01
				—8	+	'01	+	'01
ee'2	1	2	0	2	—	'01	—	'01
				0	—	1'17	—	1'18	...	+ '01
				—2	—	7'43	—	7'44	+ '02	—'01
				—4	—	'31	—	'31
				—6	—	'01	—'01
				—8	—	'01	—'01
				—10	—	'01	—'01
I-2	0	4	+	'02	+	'02
				2	+	'76	+	'76
				0	+	2'59	+	2'59	—'01	+ '01
				—2	+	2'54	+	2'54	—'01	+ '01
				—4	+	'02	+	'02
				—6	+	'02	+	'02
				—8	+	'02	+	'02
e'3	0	3	0	0	—	'10	—	'08	...	—'02
				—2	—	'35	—	'34	...	—'01
				—4	—	'01	—	'01
ey2	1	0	2	4	—	'02	—	'02
				2	—	'99	—	'99
				0	—	45'10	—	45'09	+ '01	—'02
				—2	—	'18	—	'18
				—4	—	'30	—	'22	...	—'08
				—6	—	'07	—	'03	...	—'04
				—8	—	'07	—	'03	...	—'04
I	0	—2	4	—	'07	—	'03	...	—'04	...
				2	—	6'3	—	6'36	...	—'02
				0	+	39'53	+	39'58	—'01	—'04
				—2	+	9'37	+	9'37
				—4	+	'20	+	'20
				—6	+	'20	+	'20
				—8	+	'20	+	'20

X

True Longitude—Mean Longitude. Coefficients of Sines.

C.	I.	V.	F.	D.	B.	H.	R.	B--(H+R).		
$e'\gamma^2$	0	1	2	2	+	'07	+	'06	...	+ '01
			0		+	'42	+	'42
			-2		-	2'16	-	2'15	...	- '01
			-4		-	'01	-	'04	...	+ '03
	0	1	-2	4		'00	-	'02	...	+ '02
			2		-	1'44	-	1'55	...	+ '11
			0		+	'08	+	'08
			-2		+	'38	+	'38
			-4		+	'01	+	'01
e^2a	2	0	0	1	-	'58	-	'59	...	+ '01
			-1		+	1'75	+	1'78	- '01	- '02
			-3		+	1'22	+	1'22	- '01	+ '01
			-5		+	'06	+	'06
$ee'a$	1	1	0	3	+	'02	+	'02
			1		+	1'27	+	1'27	- '01	+ '01
			-1		+	'14	+	'17	...	- '03
			-3		+	'23	+	'23
	1	-1	0	1	-	'12	-	'13	...	+ '01
			-1		-	1'09	-	1'33	+ '01	+ '23 (c)
			-3		-	'28	-	'28
e^2a	0	2	0	1	-	'04	-	'03	...	- '01
			-1		-	'04	-	'04
			-3		-	'01	-	'01
γ^2a	0	0	2	1	+	'25	+	'25
			-1		+	'58	+	'55	...	+ '03
			-3		+	'25	+	'32	...	- '07
e^4	4	0	0	2	+	'07	+	'08	...	- '01
			0		+	1'94	+	1'94
			-2		-	'95	-	'95
			-4		+	'00	+	'03	...	- '03
			-6		-	'01	-	'01
e^3e^4	3	1	0	2	-	'03	-	'02	...	- '01
			0		-	'55	-	'55
			-2		-	'48	-	'48
			-4		-	'10	-	'07	...	- '03
			-6		-	'04	-	'04
	3	-1	0	2	+	'09	+	'10	...	- '01
			0		+	'68	+	'67	...	+ '01
			-2		-	'18	-	'18
			-4		-	'03	-	'04	...	+ '01
			-6		+	'01	+ '01

True Longitude—Mean Longitude. Coefficients of Sines.

O.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).	
$e^2e'^2$	2	2	0	0	—	''07	—	'06	... —'01
			-2	—	'30	—	'28	... —'02	
			-4	—	'16	—	'16	
			-6	—	'01	—'01	
	2-2	0	2	+	'06	+	'06	
			0	+	'20	+	'19	... + 01	
			-2	+	'26	+	'23	... + '03	
			-4	+	'04	+	'03	... + '01	
ee'^3	1	3	0	0	—	'02	—	'02
			-2	—	'25	—	'26	... + '01	
			-4	—	'02	—	'02	
	1-3	0	2	+	'03	+	'03	
			0	+	'05	+	'05	
e'^4	0	4	0-2	—	'01	—'01	
$e^2\gamma^2$	2	0	2	2	—	'12	—	'12
			0	—	4'00	—	4'00	
			-2	+	'56	+	'56	
			-4	—	'01	—'01	
	2	0-2	4	—	'01	—	'01	
			2	—	'46	—	'43	... —'03	
			0	—	1'30	—	1'09	... —'21 (c)	
			-2	+	'54	+	'54	
			-4	+	'17	+	'17	
			-6	+	'01	+ '01	
$ee'\gamma^2$	1		2	2	+	'01	+	'02	... —'01
			0	+	'26	+	'27	... —'01	
			-2	+	'06	+	'06	
			-4	—	'02	—	'03	... + '01	
	1	1-2	2	+	'08	+	'08	
			0	—	'08	—	'08	
			-2	+	'43	+	'43	
			-4	+	'02	+	'02	
	1-1	2	2	—	'06	—	'08	... + '02	
			0	—	'30	—	'30	
			-2	...	'00	+	'01	... —'01	
			-4	+	'02	+	'03	... —'01	
	1-1-2	4	—	...	'01	—'01	
			2	—	'37	—	'40	... + '03	
			0	+	'08	+	'09	... —'01	
			-2	—	'07	—	'07	

X 2

True Longitude—Mean Longitude. Coefficients of Sines.

O.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).
$e'2\gamma^2$	0	2	2-2	-	'07	-	'07	...
	0	2-2	2	-	'03	-	'01	... -'02
		0			'00
		-2	+		'02	+	'02	...
γ^4	0	0	4	2	'01	+	'01	...
		0	+		'42	+	'42	...
		-2	+		'07	+	'08	... -'01
$e3a$	3	0	0	1	'04	-	'04	...
		-1	+		'13	+	'12	... + '01
		-3	+		'05	+	'06	... -'01
		-5	+		'02 + '02
$e2e'a$	2	1	0	1	'09	+	'09	...
		-1	+		'01	+	'01	...
		-3	+		'08	+	'08	...
		-5	+		'01	+	'01	...
	2-1	0	1	-	'01	-	'02	... + '01
		-1	-		'35	-	'38	... + '03
		-3	+		'04	+	'04	...
$ee'a_1$	1	2	0	1	'01		'00	... -'01
		-1			'00
		-3	+		'01	+	'01	...
	1-2	0-1			'00	-	'01	... + '01
$e\gamma^2a$	1	0	2	1	'05	+	'04	... + '01
		-1	+		'02	+	'01	... + '01
		-3	+		'03 + '03
	1	0-2	3	-	'01	-	'01	...
		1	-		'04	-	'08	... + '04
		-1	-		'02	-	'02	...
		-3	-		'01	-	'01	...
$e'\gamma^2a$	0	1	2	1	'04	-	'04	...
		-1	+		'01 + '01
		-3	+		'02 + '02
	0	1-2	3	+	'01 + '01
		1			'00	-	'06	... + '06
$e5$	5	0	0	2	'01 + '01
		0	+		'11	+	'11	...
		-2	...		'07	-	'07	...
		-4			'00	+	'01	... -'01

True Longitude—Mean Longitude. Coefficients of Sines.

O.	1.	P.	F.	D.	B.	H.	R.	B-(H+R).		
e4e'	4	1	0	0	-	'04	-	'04
			-2	-	'03	-	'03
	4-1	0	2	+	'01	+	'01
		0		+	'05	+	'05
		-2		-	'02	-	'02
e3e'2	3	2	0-2	-	'02	-	'02
		-4		-	'01	-	'01
	3-2	0	2	+	'01	+	'01
		0		+	'02	+	'01	...	+	'01
		-2		+	'01	+	'01
e2e'3	2	3	0-2	-	'01	-	'01
		-4		-	'01	-	'01
e3γ2.	3	0	2	2	-	'01	-	'01
		0		-	'33	-	'33
		-2		+	'09	+	'10	...	-	'01
	3	0-2	2	-	'03	-	'03
		0		-	'06	-	'07	...	+	'01
		-2		-	'01	-	'01
		-4		+	'01	+	'01
e2e'γ2	2	1	2	0	+	'04	+	'04
		-2		+	'03	+	'03
	2	1-2	2	+	'01	+	'01
		0		+	'03	-	'07	...	+	'10
		-2		+	'02	+	'02
		-4		+	'02	+	'01	...	+	'01
	2-1	2	2	-	'01	-	'01
		0		-	'05	-	'05
		-2			'00	+	'01	...	-	'01
	2-1-2	2		-	'03	-	'03
		0		-	'02	-	'01	...	-	'01
ee'2γ2	1	2	2-2		'00	+	'01	...	-	'01
	1	2-2-2		+	'02	+	'02
	1-2	2	0	-	'01	-	'01
		-2		+	'01	+	'01
	1-2-2	2		-	'02	+	'01	...	-	'03
		0			'00
		-2		-	'01	-	'01

True Longitude—Mean Longitude. Coefficients of Sines.

O.	L.	V.	F.	D.		B.		H.		R.	B-(H+R).
<i>ey</i> ⁴	1	0	4	0	+	"09	+	"09	
				-2	+	"01	+	"01	
	1	0	-4	2		"00	+	"03	-"03
				0	-	"08	-	"08	
				-2	-	"02	-	"02	
<i>e4a</i>	4	0	0	-1	+	"01	+	"01	
<i>e3e'a</i>	3	1	0	1	+	"01	+	"01	
	3	-1	0	-1	-	"02	-	"02	
				-3	+	"01		+"01
<i>e2γ2a</i>	2	0	2	1	+	"01		+"01
	2	0	-2	1		"00	-	"01	+"01
<i>ee'γ2a</i>	1	1	2	1	-	"01	-	"01	
	1	1	-2	1		"00	-	"05	+"05
<i>e6</i>	6	0	0	0	+	"01	+	"01	
<i>e4γ2</i>	4	0	2	0	-	"03	-	"03	
				-2	+	"01	+	"01	
	4	0	-2	0	-	"01		-"01
<i>e2γ4</i>	2	0	4	0	+	"01	+	"01	
	2	0	-4	0		"00	+	"01	-"01
<i>e3e'γ2</i>	3	-1	2	0		Not cal.	-	"01	

Latitude. Coefficients of Sines.

O.	L.	V.	F.	D.		B.		H.		R.	B-(H+R).
<i>γ</i>	0	0	1	6	+	"02	+	"01	+"01
				4	+	1'19	+	1'19	
				2	+	117'26	+	117'26	- '01	+	+"01
				0	+	18461'48	+	18463'25	- 1'77	...	
				-2	-	623'66	-	623'70	+	+"06	-"02
				-4	-	3'68	-	3'68	
				-6	-	"04	-	"04	
<i>γe</i>	1	0	1	4	+	"21	+	"21	
				2	+	15'12	+	15'12	
				0	+	1010'18	+	1010'17	- '12	+	+"13 (f)
				-2	-	166'58	-	166'60	+	+"02	...
				-4	-	6'58	-	6'58	
				-6	-	"10	-	"09	-"01
<i>-I</i>	0	1	6	+	"04	+	"04	
				4	+	3'00	+	3'00	
				2	+	199'49	+	199'48	- '02	+	+"03 (f)
				0	-	999'70	-	999'69	+	+"12	-"13 (f)
				-2	-	33'36	-	33'37	+"01
				-4	-	"48	-	"47	-"01
				-6	-	"01		"00	-"01

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Latitude. Coefficients of Sines.

O.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).		
$\gamma e'$	0	1	1	4	-	'02	-	'03	...	+ '01
				2	-	1'27	-	1'28	...	+ '01
				0	-	6'49	-	6'50	+ '01	...
				-2	-	29'69	-	29'74	+ '04	+ '01
				-4	-	'42	-	'41	...	- '01
				-6	-	'01	- '01
	0-1	1	1	4	+	'15	+	'16	...	- '01
				2	+	8'00	+	8'00	- '01	+ '01
				0	+	4'86	+	4'88	- '01	- '01
				-2	+	12'14	+	12'14	- '01	+ '01
				-4	+	'11	+	'10	...	+ '01
γa	0	0	1	3	-	'03	-	'03
				1	-	5'36	-	5'25	+ '04	- '15
				-1	+	4'80	+	4'69	- '04	+ '15
				-3	+	'35	+	'35
$\gamma 3$	0	0	3	2	-	'14	-	'15	...	+ '01
				0	-	6'30	-	6'30
				-2	-	2'19	-	2'19
				-4	-	'06	-	'07	...	+ '01
γe^2	2	0	1	4	+	'03	+	'03
				2	+	1'52	+	1'52
				0	+	61'91	+	61'90	- '01	+ '02
				-2	-	15'57	-	15'56	...	- '01
				-4	-	'64	-	'63	...	- '01
				-6	-	'08	-	'07	...	- '01
	-2	0	1	6	+	'06	+	'06
				4	+	2'41	+	2'42	...	- '01
				2	-	1'62	-	1'62
				0	-	31'76	-	31'77	...	+ '01
				-2	-	2'15	-	2'15
				-4	-	'05	-	'05
γe^3	1	1	1	4	-	'01	-	'01
				2	-	'24	-	'24
				0	-	5'33	-	5'34	+ '01	...
				-2	-	7'46	-	7'48	+ '01	+ '01
				-4	-	'60	-	'52	...	- '08
				-6	-	'02	-	'00	...	- '02

Latitude. Coefficients of Sines.

O.	L.	U.	F.	D.	B.	H.	R.	B-(H+B).
yea'	-1-1	1	6	+	'01	+	'01	...
			4	+	'34	+	'35	...
			2	+	8'90	+	8'91	...
			0	+	5'10	+	5'13	...
			-2	+	'83	+	'83	...
			-4	+	'02	+	'01	...
	1-1	1	4	+	'03	+	'03	...
			2	+	1'14	+	1'14	...
			0	+	6'76	+	6'76	...
			-2	+	'80	+	'80	...
			-4	+	'17	+	'15	...
	-1	1	4	-	'05	-	'06	...
			2	-	1'32	-	1'32	...
			0	-	5'66	-	5'67	...
			-2	-	1'77	-	1'78	...
			-4	-	'06	-	'05	...
yea'	0	2	1	2	'02	-	'02	...
			0	-	'06	-	'06	...
			-2	-	1'10	-	1'10	...
			-4	-	'03	-	'02	...
	0-2	1	4	+	'01	+	'01	...
			2	+	'39	+	'39	...
			0	+	'02	+	'02	...
			-2	+	'14	+	'13	...
yea	1	0	1	3	'01	-	'01	...
			1	-	'67	-	'65	...
			-1	+	'43	+	'42	...
			-3	+	'31	+	'30	...
			-5	+	'01	-	'00	...
	-1	0	1	3	'21	-	'21	...
			1	+	'14	+	'13	...
			-1	+	'59	+	'58	...
			-3	+	'04	+	'04	...
yea'd	0	1	1	3	'01	+	'01	...
			1	+	'80	+	'79	...
			-1	+	'01	+	'02	...
			-3	+	'03	-	'01	...
	0-1	1	1	-	'02	-	'01	...
			-1	-	'81	-	'79	...
			-3	-	'04	-	'02	...

Latitude. Coefficients of Sines.

O.	I.	V.	F.	D.	B.	H.	R.	B-(H+R).		
736	1	0	3	2	-	'03	-	'03
				0	-	1'02	-	1'02
				-2	-	'33	-	'33
				-4	+	'01	+	'02	...	- '01
	-1	0	3	4	-	'01	-	'01
				2	-	'24	-	'25	...	+ '01
				0	-	2'81	-	2'81
				-2	+	'29	+	'29
				-4	+	'01		'00	...	+ '01
736'	0	1	3	0	+	'01	+	'01
				-2	-	'09	-	'09
				-4	-	'01		'00	...	- '01
	0-1		3	2	-	'01	-	'01
				0		'00		'00
				-2	+	'06	+	'07	...	- '01
743	3	0	1	2	+	'14	+	'14
				0	+	3'98	+	3'98
				-2	-	1'52	-	1'52
				-4	+	'01		'00	...	+ '01
				-6	-	'01		'00	...	- '01
	-3	0	1	6	+	'03	+	'03
				4	+	'02	+	'02
				2	+	'26	+	'27	...	- '01
				0	-	1'59	-	1'59
				-2	-	'15	-	'15
7426'	2	1	1	2	-	'03	-	'03
				0	-	'64	-	'64
				-2	-	'66	-	'66
				-4	-	'05	-	'03	...	- '02
				-6	-	'01	- '01
	-2-1		1	6	+	'01	+	'01
				4	+	'22	+	'22
				2	-	'06	-	'06
				0	+	31	+	31
				-2	+	'06	+	'06
	2-1		1	2	+	'11	+	'13	...	- '02
				0	+	'81	+	'80	...	+ '01
				-2	-	'08	-	'09	...	+ '01
	-2	1	1	4	-	'03	-	'03
				2	+	'06	+	'06
				0	-	'30	-	'31	...	+ '01
				-2	-	'13	-	'12	...	- '01
				-4	-	'01	- '01

Latitude. Coefficients of Sines.

O.	I.	V.	F.	D.	B.	H.	R.	B-(H+R).			
$\gamma_{ee}^{1/2}$	1	2	1	0	—	'06	—	'06	
			-2	—	'27	—	'28	...	+	'01	
			-4	—	'03	—	'03	
	-1-2	1	4	+	'02	+	'02	
			2	+	'32	+	'32	
			0	+	'06	+	'06	
			-2	+	'01	+	'01	
	1-2	1	2	+	'05	+	'06	...	—	'01	
			0	+	'12	+	'12	
			-2	+	'11	+	'10	...	+	'01	
	-1	2	1	2	—	'12	—	'12	
			0	—	'10	—	'11	...	+	'01	
			-2	—	'07	—	'06	...	—	'01	
$\gamma_{e'}^{1/3}$	0	3	1-2	—	'04	—	'01	...	—	'03	
	0-3	1	2	+	'01	+	'02	...	—	'01	
γ_{3a}	0	0	3	1	+	'01	+	'01	
			-1	+	'03	+	'03	
			-3	+	'01	+	'02	...	—	'01	
γ_{e2a}	2	0	1	1	—	'07	—	'06	...	—	'01
			-1	+	'11	+	'11	
			-3	+	'04	+	'05	...	—	'01	
			-5	+	'01	—	'00	...	+	'01	
	-2	0	1	5	—	'01	—	'01	
			3	—	'05	—	'05	
			1	—	'08	—	'07	...	—	'01	
			-1	+	'04	+	'03	...	+	'01	
$\gamma_{ee}^{1/4}$	1	1	1	1	+	'10	+	'10	
			-1	—	'01	—	'02	...	+	'01	
			-3	+	'02	+	'01	...	+	'01	
	-1-1	1	3	—	'01	—	'01	
			1	—	'00	—	'01	...	+	'01	
			-1	—	'03	—	'01	...	—	'02	
	1-1	1	1	—	'01	—	'01	
			-1	—	'01	—	'02	...	+	'01	
			-3	—	'01	—	'00	...	—	'01	
	-1	1	1	3	+	'02	+	'02	
			1	—	'06	—	'05	...	—	'01	
			-1	+	'02	+	'01	...	+	'01	
γ_5	0	0	5	0	+	'01	+	'01	

Latitude. Coefficients of Sines.

G.	L.	U.	F.	D.	B.	H.	R.	B-(H+R).
γ_{382}	2	0	3	0	-	'12
			-2	-	'02
			-4	+	'01	+	'01	...
	2	0	3	4	-	'01
			2	-	'07	-	'07	...
			0	+	'13	+	'11	... + '02
			-2	+	'01	+	'01	...
γ_{383}	1	1	3	0	+	'01
			-2	-	'01	-	'01	...
	-1	-1	3	2	-	'01
			0	+	'01		'00	... + '01
			-2	-	'01	-	'01	...
	1	-1	3	0	-	'01
	-1	1	3	0	-	'01
			-2	+	'02	+	'03	... - '01
γ_{41}	4	0	1	2	+	'01
			0	+	'27	+	'26	... + '01
			-2	-	'14	-	'13	... - '01
			-4	+	'01	+	'01	...
	-4	0	1	2	+	'03
			0	-	'09	-	'09	...
			-2	-	'01	-	'01	...
γ_{436}	3	1	1	0	-	'06
			-2	-	'06	-	'06	...
			-4		'00	+	'01	... - '01
	-3	-1	1	2	+	'01
			0	+	'02	+	'03	... - '01
			-2	+	'01 + '01
	3	-1	1	2	+	'01
			0	+	'08	+	'08	...
			-2	-	'02	-	'03	... + '01
	-3	1	1	4		'00 - '01
			0	-	'02	-	'02	...
			-2	-	'01 - '01
γ_{426}^2	2	2	1	0	-	'01
			-2	-	'02	-	'03	... + '01
	-2	-2	1	4	+	'01
	2	-2	1	2	+	'01
			0	+	'02	+	'02	...
			-2	+	'01	+	'02	... - '01
	-2	2	1	2	-	'01
			0	-	'01		'00	... - '01
			-2	-	'01 - '01

Latitude. Coefficients of Sines.

O.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).
$\gamma e e'3$	1	3	1	-2	-	'01
	-1	-3	1	2	+	'01
$\gamma e3a$	3	0	1	1	-	'01
			-1		+	'01
	-3	0	1	1	-	'01
$\gamma e2'e'a$	2	1	1	1	+	'01
	2	-1	1	-1	-	'02
	-2	1	1	1	+	'02
$\gamma3e3$	3	0	3	0	-	'01
	-3	0	3	0	+	'00
$\gamma e5'$	5	0	1	0	+	'02
			-2		-	'01
	-5	0	1	0	-	'01
$\gamma e4e'$	4	-1	1	0	Not cal.	'01
	4	1	-1	-2	"	'01

Sine Parallax. Coefficients of Cosines.

O.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).				
I	0	0	0	6	+	'003	+	'002	+	'001		
				4	+	'261	+	'261		
				2	+	28'233	+	28'225	+	'005	+	'003
				0	+	3422'700	+	3422'09	+	'61		
e	1	0	0	6	+	'001	+	'001	
				4	+	'043	+	'043		
				2	+	3'086	+	3'084	+	'001	+	'001
				0	+	186'540	+	186'483	+	'029	+	'028
				-2	+	34'312	+	34'309	+	'005	-	'002
				-4	+	'601	+	'599	+	'002
				-6	+	'009	+	'007	+	'002
e'	0	1	0	4	-	'005	-	'004	-	'001
				2	-	'300	-	'301	+	'001
				0	-	'400	-	'393	-	'007
				-2	+	1'920	+	1'920	-	'002	+	'002
				-4	+	'034	+	'035	-	'001
				-6	+	'001	+	'001
a	0	0	0	3	+	'002	+	'003	-	'001
				1	-	'975	-	'953	+	'007	-	'029
e ²	2	0	0	4	+	'005	+	'004	+	'001
				2	+	'283	+	'283
				0	+	10'166	+	10'161	+	'002	+	'003
				-2	-	'304	-	'302	-	'002
				-4	+	'372	+	'372
				-6	+	'011	+	'010	+	'001

Sine Parallax. Coefficients of Cosines.

O.	I.	P.	F.	D.	B.	H.	R.	B-(H+R).
<i>ee'</i>	I	I	O	4	—	—'001
			2	—	'049	—	'049	...
			0	—	'950	—	'961	+ '001
			—2	+	I'446	+	I'447	—'002
			—4	+	'067	+	'069	...
			—6	+	'002	+	'001	...
	I—I	O	4	+	'006	+	'007	...
			2	+	'231	+	'229	...
			0	+	I'154	+	I'144	—'001
			—2	—	'226	—	'227	...
			—4	—	'010	—	'009	...
			—6	—	'001	—'001
<i>e'z</i>	O	2	O	2	—	'003	—	'002
			0	—	'009	—	'008	...
			—2	+	'092	+	'092	...
			—4	+	'003	+	'002	...
<i>γ^z</i>	O	O	2	2	—	'001	—	'001
			0	—	'012	—	'012	...
			—2	—	'105	—	'105	...
			—4	+	'003	+	'002	...
<i>ea</i>	I	O	O	I	—	'109	—	'106
			—1	+	'012	+	'011	...
			—3	—	'039	—	'037	...
<i>e'a</i>	O	I	O	3	+	'003	+	'002
			I	+	'149	+	'146	—'001
			—1	—	'004	—	'004	...
			—3	+	'001	+	'001	...
<i>e³</i>	3	O	O	4	+	'001
			2	+	'024	+	'023	...
			0	+	'622	+	'620	...
			—2	—	'119	—	'121	...
			—4	+	'007	+	'008	...
			—6	+	'005	+	'004	...
<i>e^{2e'}</i>	2	I	O	2	—	'005	—	'004
			0	—	'104	—	'122	...
			—2	—	'019	—	'019	...
			—4	+	'032	+	'032	...
			—6	+	'002	+	'001	...
	2—I	O	4	+	'001
			2	+	'021	+	'022	...
			0	+	'127	+	'149	...
			—2	—	'002	—	'002	...
			—4	—	'004	—	'004	...

Sine Parallax. Coefficients of Cosines.

O.	L.	P.	F.	D.	B.	H.	R.	B-(H+R).
ee^2	1	2	0	0	-.011	-.010	...	-.001
			-2	+	.049	.049
			-4	+	.004	.004
	1-2	0	4	+	.001	+.001
			2	+	.011	.012	...	-.001
			0	+	.020	.012	...	+.008
			-2	-	.021	.021
e^3	0	3	0-2	+	.004	.004
$e\gamma^2$	1	0	2	0	-.001	.000	...	-.001
			-2	-	.083	.083
			-4	+	.001	.001
	1	0-2	4	-	.001	-.001
			2	-	.048	.048
			0	-	.714	.709	...	-.005
			-2	-	.011	.011
$e^1\gamma^2$	0	1	2	0	.001	.001
			-2	-	.007	.007
			-4	+	.001	+.001
	0	1-2	2	+	.001	.001
			0	+	.002	.001	...	+.001
e^2a	2	0	0	1	-.010	.010
			-1	+	.016	.015	...	+.001
			-3	-	.009	.009
			-5	-	.001	-.001
ee^1a	1	1	0	1	.016	.016
			-1	-	.000
			-3	-	.003	.002	...	-.001
	1-1	0	1	-	.001	.001
			-1	-	.000	.001	...	+.001
			-3	+	.004	.003	...	+.001
γ^2a	0	0	2-1	+	.007	.007
			-3	-	.002	.002
e^4	4	0	0	2	.002	.001	...	+.001
			0	+	.040	.040
			-2	-	.013	.014	...	+.001
e^3e^1	3	1	0	2	-.001	-.001
			0	-	.010	.010
			-2	-	.005	.004	...	-.001
			-4	+	.001	+.001
			-6	+	.001	+.001
	3-1	0	2	+	.002	.001	...	+.001
			0	+	.012	.012
			-2	-	.002	.001	...	-.001

Sine Parallax. Coefficients of Cosines.

O.	I.	V.	F.	D.	B.	H.	R.	B-(H+R).
$e^2e'^2$	2	2	0	0	-	'001	...	- '001
			-2		-	'001
			-4		+	'002
	2-2	0	2		+	'001
			0		+	'002	...	+ '001
			-2			'000	...	- '001
			-4		-	'001	...	- '001
ee'^3	1	3	0-2		+	'001
$e^2\gamma^2$	2	0	2-2		-	'009
	2	0-2	2		-	'005	...	- '001
			0			'000
			-2		-	'014
$ee'\gamma^2$	1	1	2-2		-	'003	...	- '001
	1	1-2	2		+	'001	...	+ '001
			0		+	'002
			-2		-	'001	...	- '001
	1-1	2-2				'000	...	- '001
	1-1-2	2			-	'003
			0		-	'003
e^3a	3	0	0	1	-	'001
			-1		+	'002	...	+ '001
$e^2e'a$	2	1	0	1	+	'002	...	+ '001
			-1			'000
			-3		-	'001	...	- '001
	2-1	0	1		-	'001	...	- '001
			-1		-	'003
			-3		+	'001	...	- '001
ek^2a	1	0	2-1		+	'001	...	+ '001
	1	0-2-1			+	'001	...	+ '001
e^5	5	0	0	0	+	'003
			-2		-	'001	...	+ '001
e^4e'	4	1	0	0	Not cal,	-	'001	...
			-2		"	-	'001	...
	4-1	0	0		"	+	'001	...
e^3k^2	3	0	2-2		-	'001
	3	0-2	2		-	'001	...	- '001
			-4		-	'001	...	- '001

Postscript added 1905 January 25.—The coefficients resulting from the theory are given above to hundredths of a second in longitude and latitude, and to thousandths of a second in parallax. In each case they have actually been calculated to one more place of decimals. Further, the coefficients, referred to rectangular coordinates, have been found with even greater accuracy in the great majority of cases. The final results with the full number of places of decimals used are probably correct within two units of the last figure given, as far as the terms in any one characteristic are concerned; they will be found in the fourth part of the "Theory of the Motion of the Moon," shortly to appear in the *Memoirs* of the Society.

The coefficients are all definitive with the exception of two or three, which require small corrections not exceeding $0''.02$, due to certain terms of the disturbing function which have been neglected. See *Mem. R.A.S.* vol. liii. p. 50.

Haverford College:

1904 November 15.